INDIAN STATISTICAL INSTITUTE
BANGALORE CENTRE

BACKPAPER EXAMINATION
M. MATH, II YEAR, II SEMESTER, 2014-15
ERGODIC THEORY

Time limit: 3 hours   Maximum marks: 100

1. State and prove Poincare's Recurrence Theorem for a measure preserving trnasformation. [10]

2. Let μ be a Borel probability measure on S^1 which is invariant under all rotations. Prove that $\int z^n d\mu(z) = 0$ if $n \neq 0$. Prove that there is only one such probability measure. Do not use the theorem on uniqueness of Haar measure. [15]

3. Let $Tz = z^2, z \in S^1$. Let $\mathcal{A}$ be the field consisting of the right half semi-circle and the left half semi-circle. Compute $h(\mathcal{A}, T)$. [20]

4. Recall the construction of Kakutani Tower in Rokhlin's Lemma: $\epsilon > 0$ and $N \in \mathbb{N}$ are given and we choose B with $0 < P(B) < \frac{\epsilon}{N}$; $B_j = \{\omega \in B : T\omega \notin B, T^2\omega \notin B, ..., T^{j-1}\omega \notin B, T^j\omega \in B\}$ and the sets in the tower are $T^i(B_j) : 0 \leq i \leq j-1, j = 1, 2, ...$ Prove that any two sets in the tower are disjoint. [15]

5. Let T be an ergodic measure preserving transformations with respect to two probability measures P and Q on $(\Omega, \mathcal{F})$. Show that either $P = Q$ or P is singular with respect to Q. [15]

6. Let $(\Omega, \mathcal{F}, P, T)$ be a dynamical system, $1 < p < \infty$ and $f \in L^p$. Show that $\{\frac{1}{n}\sum_{k=0}^{n-1} f \circ T^k\}$ converges in L^p $\forall f \in L^p$. [15]

7. Is the map $z \to \frac{1}{z}$ on S^1 measure preserving? Is it ergodic? [10]